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CHARGING OF WEAKLY CONDUCTING PARTICLES OF A SUSPENSION ON COLLISION WITH BOUNDING SURFACES*

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The change of electric charge of weakly conducting particles in flowing suspension on their collision with walls is determined. A boundary condition for electric current density in rarefied suspensions of weakly conducting particles in nonconducting gas at impenetrable walls is obtained.

1. Boundary condition for the electric current density at impenetrable walls. In investigations of charged particle motion within the frame work of continuous medium mechanics /1,2/ it is necessary to specify the boundary condition for the electric current density. Let us consider the derivation of that condition for a rarefied suspension consisting of a nonconducting gas and electrically charged particles whose conductance although fairly small (in the sense indicated below), is nonzero.

It was shown in /3,4/ that, as the result of suspended particles collision with a solid wall, their charges may change. In particular, initially uncharged particles may become charged, i.e. electrified. When the change Δe_p of the electric charge of one particle is known, it is possible to establish the boundary condition for the electric current density $\mathbf{j}$ of the form

$$(\mathbf{j}\mathbf{v}) = -\int g(\mathbf{v})\,\Delta e_p(\mathbf{v}\mathbf{v})\,d\mathbf{v} \tag{1.1}$$

where $\mathbf{v}$ is the outward normal to the wall that represents the bounding surface, $\mathbf{v}$ is the particle velocity, and $g(\mathbf{v})$ is a function of velocity distribution of suspension particles impinging on the wall (for which $(\mathbf{v}\,\mathbf{v}) < 0$). In the absence of scatter of velocities of particles impinging on the wall we have $g(\mathbf{v}) = n^-\delta(\mathbf{v} - \mathbf{v}^-)$, where n^- and $\mathbf{v}^-$ are the concentration and velocity of particles in the stream impinging on the wall. When the effect of the electric field on the motion of suspension particles can be neglected, function $g(\mathbf{v})$ is determined by solving a purely mechanical problem. If, furthermore, the velocities of particles impinging on the wall are uniform, it is sufficient to determine n^- and $\mathbf{v}^-$ for obtaining $g(\mathbf{v})$.

Let us consider the change Δe_p of electric charge of a single particle at its impact on a metal wall, on the usual assumption that the duration of contact of particle and wall is considerably longer than the relaxation time of the potential of the latter. It is therefore possible to assume in the calculation of Δe_p that the wall potential is constant (below we assume it to be zero).

2. Distribution of the electric charge and field in a particle prior to its impact on the wall. Let the particle conductance be determined by the presence in it of N varieties of electric charge carriers, such as ions or, in the case of semiconductors particles, electron and vacancies /5,6/. We assume the concentration of charge carriers to be fairly small so that their mobility and diffusion coefficients can be linked by Einstein's relations /5,6/, and that the particle conductance σ satisfies the formulas (in Gaussian system of units)

$$\sigma = \sum \frac{e_i^2 n_i^\circ D_i}{kT}, \quad \frac{\varepsilon_p D}{4\pi\sigma} \sim d^2 = \frac{\varepsilon_p kT}{4\pi \sum e_i^2 n_i^\circ} \tag{2.1}$$

where e_i, D_i, and n_i° are, respectively, the charge, the diffusion coefficient, and the characteristic concentration of charge carriers of the i variety, D is the characteristic value of coefficients D_i, T is the absolute temperature, k is the Boltzmann constant, and ε_p and d are, respectively, the permittivity and the Debye radius of particle material.

Let $\mathbf{E}_0$ be the electric field intensity at the wall in the absence impinging particles. In rarefied suspension $\mathbf{E}_0$ usually coincides with the electric field intensity defined by averaging over a physically infinitely small volume containing a fairly large number of suspended particles. Let us determine the concentration of charge carriers at the surface of a spherical particle carrying the over-all electric charge e_p immediately before its collision with the wall under the condition

$$\frac{R}{v} \ll \frac{\varepsilon_p}{4\pi\sigma} \sim \frac{d^2}{D} \sim \tau_e, \quad \tau_e \ll \frac{l_E}{v}, \quad \tau_e \ll \tau_E, \quad d \ll R \tag{2.2}$$

where v is the absolute velocity of a particle approaching the wall, R is the particle radius, and l_E and τ_E are, respectively, the characteristic length and time of change of the electric

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field mean intensity in the suspension.

The first of inequalities (2.2) implies that the time a particle remains close to the wall ($\sim R/v$) is considerably shorter than the relaxation time τ_e of the electric charge distribution in the particle. Hence it is possible to disregard the effect of the wall on the charge carrier distribution in the particle prior to collision. The second and third of equalities (2.2) show that the relaxation of the electric charge in a particle is many times faster than the change of electric field intensity which acts on the particle. It is, thus, possible for the charge carrier distribution in the particle immediately before collision to be the same as in the electrostatic problem of the charged particle in a constant uniform external field of intensity $\mathbf{E}_0$ in the absence of a wall.

Note that the first and second of conditions (2.2) imply that $l_E \gg v\tau_e \gg R$ and, consequently, the electric field intensity $\mathbf{E}$ along the section of particle trajectory of length of the order of $v\tau_e \gg R$ preceding the impact differs only little from the electric field intensity $\mathbf{E}_0$ directly at the wall in the absence of a particle; for an infinite flat wall $\mathbf{E} = \mathbf{E}_0$ in the absence of any other bodies. The fourth of inequalities (2.2) implies that almost the total electric charge is concentrated in the thin surface layer of thickness $\sim d$.

When determining the concentration n_{is} of charge carriers on the surface of a particle, it is admissible to set $R = \infty$ and use the formulas for the half-space /5,6/

$$n_{is} = n_i^\circ \exp\left(-\frac{e_i\psi(0)}{kT}\right), \quad \frac{d^2\psi}{dz^2} = -\frac{4\pi}{\varepsilon_p}\sum e_i n_i^\circ\left[\exp\left(-\frac{e_i\psi}{kT}\right) - 1\right] \tag{2.3}$$

$$\psi(\infty) = 0, \quad -\varepsilon_p\psi'(0) = \varepsilon(\mathbf{E}_s'\boldsymbol{\nu}) \equiv \varepsilon E' \tag{2.4}$$

where ε is the permittivity of the carrier phase, $\mathbf{E}_s'$ is the electric field intensity on the outside surface of the particle in the previously mentioned electrostatic problem, and $\boldsymbol{\nu}$ is the inward normal to the particle surface. Vectors $\mathbf{E}_s'$ and $\boldsymbol{\nu}$ are obviously assumed to be at the same point of particle surface at which n_{is} is calculated. Function $\psi(z)$ which is completely determined by the second of Eqs. (2.3) and boundary conditions (2.4) may be taken as the distribution of electric potential in the half-space ($z > 0$) at whose surface (at $z \leqslant 0$) the electric field intensity is E' and (when $E' > 0$) is oriented along the z-axis to the half-space inside. For a spherical particle with $d \ll R$ the quantity E' defined above is readily obtained with the use of the formula for perfectly conducting particles /7/ (for the same R, e_p, and $\mathbf{E}_0$) and is related to the surface charge density q on it by the formula $4\pi q = -\varepsilon E'$. At the contact point of particle and wall

$$E' = 3E_0 - e_p/(\varepsilon R^2) \tag{2.5}$$

The second of Eqs. (2.3) has a first integral which in conformity with the first of conditions (2.4) can be written thus:

$$\frac{1}{2}\left(\frac{d\psi}{dz}\right)^2 = \frac{4\pi}{\varepsilon_p}\sum e_i n_i^\circ\left[\frac{kT}{e_i}\left[\exp\left(-\frac{e_i\psi}{kT}\right) - 1\right] + \psi\right] \tag{2.6}$$

Formula (2.6) with allowance for the second of conditions (2.4) enables us to determine the dependence of $\psi(0)$ on E' which at the contact point of particle and wall is elementary defined in terms of E_0 and e_p with the use of equality (2.5). Then the first of equalities (2.3) determines the charge carrier concentration at the contact point of the particle surface in terms of E_0 and e_p.

Let us consider two examples.

Example 1. Let the particles represent a semiconductor with electron or vacany extrinsic conductance (charge carriers are electrons or positively charged vacancies). The implicit dependence of $\psi(0)$ on E' is of the form

$$\frac{1}{2}\left(\frac{\varepsilon}{\varepsilon_p}E'\right)^2 = \frac{4\pi n^\circ kT}{\varepsilon_p}\left[\exp\left(\mp\frac{e\psi(0)}{kT}\right) - 1 \pm \frac{e\psi(0)}{kT}\right] \tag{2.7}$$

where e is the electron charge ($e < 0$), n° is the density of electrons or vacancies of conductance in a particle when $e_p = E_0 = 0$, and the upper sign corresponds to electron conductance and the lower to that of vacancies. If $|e\psi| \ll kT$, formula (2.7) is readily solved for $\psi(0)$ by substituting a linear function for the exponent. As the result, we obtain for $\psi(0)$ and electron or vacancy concentrations $n_{\mp s}$ on the particle surface the following formulas:

$$\psi(0) = \frac{\varepsilon E'}{\varepsilon_p}\sqrt{\frac{\varepsilon_p kT}{4\pi e^2 n^\circ}}, \quad n_{\mp s} = n^\circ \exp\left(\mp\frac{e\psi(0)}{kT}\right) \simeq n^\circ\left(1 \mp \frac{e\psi(0)}{kT}\right)$$

where E' is determined at the contact point by formula (2.5).

Example 2. Let the particle have binary ion conductance (positive and negative ions of charge carriers) with equal absolute values of ion charges. The dependence between $\psi(0)$ and E' is then of the form

$$\frac{1}{2}\left(\frac{\varepsilon}{\varepsilon_p}E'\right)^2 = \frac{4\pi n^\circ kT}{\varepsilon_p}\left(\alpha-\frac{1}{\alpha}\right)^2, \quad \alpha \equiv \exp\frac{e\gamma\psi(0)}{2kT} \tag{2.8}$$

where n° is the concentration of positive or negative ions when $e_p = E_0 = 0$, and γ represent the charge numbers of positive and negative ions. Solving equality (2.8) for α, we obtain for it an expression in terms of E'

$$\alpha = \sqrt{1+A^2}+A, \quad A \equiv -{}^1/_4\varepsilon E'\,(2\pi\varepsilon_p n^\circ kT)^{-1/2} \tag{2.9}$$

For the determination of negative and positive ion concentration $(n_{\mp s})$ on the particle surface we now have the formula

$$n_{\mp s} = n^\circ \exp\left(\mp\frac{e\gamma\psi(0)}{kT}\right) = n^\circ\alpha^{\mp 2} = n^\circ(\sqrt{1+A^2}\mp A)^2$$

and, as implied by equalities (2.5) and (2.9), at the point of contact

$$A = {}^1/_4\,(2\pi\varepsilon_p n^\circ KT)^{-1/2}\,(e_p/R^2 - 3\varepsilon E_0)$$

Let us determine the projection on $\mathbf{v}$ of the electric field intensity E_w between the particle and wall immediately before the collision, i.e. at the point of particle and wall contact. Vector $\mathbf{v}$ then obviously coincides with the outward normal to the wall. The quantity E_w can be represented in the form of the sum

$$E_w = 2\,(E'-E_0)+E_0 \tag{2.10}$$

where $E'-E_0$ is the projection on $\mathbf{v}$ of the intensity of the electric field generated by a particle whose electric charge is distributed as in the above electrostatic problem of a particle in the external field $\mathbf{E}_0$ and carrying the charge e_p. The multiplier 2 is introduced here to account for the electric field generated by the particle electrostatic image relative to the wall surface which in the investigation of separate particles may be assumed flat. The last term in (2.10) represents the contribution to E_w of the electric field of intensity $\mathbf{E}_0$, which is external relative to the particle. Now, using for E' the expression (2.5), we represent formula (2.10) in the form

$$E_w = 5E_0 - \frac{2e_p}{\varepsilon R^2} \tag{2.11}$$

Let us also determine the projection on $\mathbf{v}$ of the electric field of intensity E_s inside the particle at the point of its contact with the wall. Since $\varepsilon_p E_s = \varepsilon E_w$, hence, using (2.11) we obtain

$$E_s = \frac{5\varepsilon}{\varepsilon_p}E_0 - \frac{2e_p}{\varepsilon_p R^2} \tag{2.12}$$

The described calculation of the electric charge distribution and field in a particle is approximate and valid only when conditions (2.2) are satisfied. Generally, the problem is to be considered in rigorous formulation which requires the solution of general equations of diffusion for charge carrier concentration and of Maxwell equations for the electric field.

3. Formula for current density for charging a single particle. Let us assume that the particle is charged at impact on a wall as the result of interaction of the latter with charge carriers of some single variety $r\,(1\leqslant r\leqslant N)$. This means that the number of carriers of the r variety in the particle may change owing to the interaction with the wall, while the number of charge carriers of any other variety $i\neq r$ remains constant.

In the above examples such situation corresponds to cases in which: 1) exchange of electrons between the wall and particle can take place (example 1 in the case of electron conductance), 2) the wall can absorb positive vacancies from the particle or impart to it such vacancies (example 1 in the case of vacancy conductance), and 3) ions of one variety may be discharged (or bound) or, conversely, ions of a new variety (positive or negative, example 2) may be generated.

Let furthermore the diffusion of charge carriers of the r variety be the restricting stage of the particle charging process. This enables us to assume that during the whole time of collision duration the relations

$$n_r = n_w = \text{const}; \quad -D_i\frac{\partial n_i}{\partial z} + \frac{e_i D_i n_i}{kT}E_z = 0, \quad i\neq r \tag{3.1}$$

are satisfied at the contact interface of particle and wall. In these formulas n_i is the concentration of charge carriers of the i variety, E_z is a component of the electric field intensity $\mathbf{E}$, and the Cartesian system of coordinates xyz has its z-axis directed along the outward normal to the wall, and the coordinate origin located at the point of contact. The constant n_w, which may depend on T and E_s, is an effective parameter representing physical properties of the wall. Because of this, the boundary condition defined by the first of equalities (3.1) is on the whole approximate. In precise formulation it is necessary to take into account also the motion of charge carriers in the wall, by specifying the conditions at the particle—wall surface at contact. If charge carriers of the r variety participate in the reaction at the wall, whose rate is infinitely high, then strictly $n_w = 0$.

Concentration n_i and the electric field potential φ satisfy the equations of electro-diffusion and the Poisson's equation

$$\frac{\partial n_i}{\partial t} + \mathrm{div}\left(-D_i \nabla n_i + \frac{e_i D_i n_i}{kT}\mathbf{E}\right) = 0, \quad \Delta\varphi = -\frac{4\pi}{\varepsilon_p}\sum e_i (n_i - n_i^\circ), \quad \mathbf{E} = -\nabla\varphi \tag{3.2}$$

where t is the time counted from the instant of impact of particle on the wall.

The kind of distribution of concentration n_i for $t \leqslant 0$ was defined in the preceding Section. It represents the equilibrium state for an isolated particle in the external field $\mathbf{E}_0$. Near the wall that distribution is evidently unstable. It changes in the course of the collision /duration/ time τ, even when the number of charge carriers of any variety is not changed by interaction with the wall, and consequently there is no charging (in that case the second of equalities (3.1) is valid for all i, including $i = r$). However the indicated variation of n_i during time τ can be neglected, since usually $\tau \lesssim R / v$ /8/. From (2.2) we have

$$\tau \ll \frac{\varepsilon}{4\pi\sigma} \sim \tau_e \tag{3.3}$$

which shows that the collision time is considerably shorter than the relaxation time of the particle electric charge. Because of this, the purely electrostatic effect of the wall on charge carrier distribution in the particle during collision is small.

The concentration n_r may also vary as the result of change of the number of charge carriers of the r variety during their interaction with the wall (the concentration n_r on the wall is, then, defined by the first of equalities (3.1)). In what follows we consider the case when at the instant of collision termination a noticeable change of concentration n_r occurs only in the boundary layer inside the particle (close to the contact surface). The thickness δ of that boundary layer satisfies the ineqalities

$$\delta \ll d, \quad \delta \ll r_c \tag{3.4}$$

where r_c is a characteristic dimension of the contact area during the particle impact on the wall. For r_c we have the estimate $r_c^2 \sim Rv\tau$ /8/. The expression for δ in terms of the problem parameters is given below.

In the problem of particle charging by the change of the number of its charge carriers of the r variety which interact with the wall during collision, the characteristic values of variables x, y, z, t and n_i are equal $r_c, r_c, \delta, \tau,$ and n_{is}. We introduce therefore the dimensionless parameters, separating the quantities

$$D_i, e_i, x, y, z, t, n_i, \varphi$$

according to their characteristic values $D \equiv D_r, e_r, r_c, r_c, \delta, \tau, n_{is},$ and kT / e_r, where the diffusion coefficient of charge carriers of the r variety is taken for D.

Representing the system of Eqs. (3.2) in dimensionless form and neglecting terms of order δ^2 / d^2 and δ^2 / r_c^2, for the concentration n_r we obtain the equations

$$\frac{\partial n^*}{\partial t^*} - \frac{D\tau}{\delta^2}\left(\frac{\partial^2 n^*}{\partial z^{*2}} + \frac{\partial\varphi^*}{\partial z^*}\frac{\partial n^*}{\partial z^*}\right) = 0, \quad \frac{\partial^2\varphi^*}{\partial z^{*2}} = 0 \tag{3.5}$$

where the subscript r is henceforth omitted, and the asterisk denotes the normalization operation. Boundary conditions for system (3.5), as $z^* \to \infty$ are the conditions of asymptotic merging of values of concentration n and electric field intensity inside the boundary layer with time independent quantities $n(z)$ and $E_z(z)$ outside the boundary layer, as $z \to 0$ (suitably normalized). The latter coincide with the concentration $n_s(E_0, e_p)$ and the electric field intensity $E_s(E_0, e_p)$ at the point of contact, which were defined in the preceding Section by equalities (2.2)—(2.4) and (2.12). Hence the following conditions at infinity:

$$n^* = 1, \; -\partial\varphi^* / \partial z^* = e_r E_s \delta / (kT), \; z^* \to \infty \tag{3.6}$$

At $z^* = 0$ the boundary conditions for system (3.5) are defined by the first of Eqs. (3.1) and the established in Sect.1 condition of zero potential of the wall. In dimensionless quantities these conditions at the wall are of the form

$$n^* = n_w / n_s, \quad \varphi^* = 0, \quad z^* = 0 \tag{3.7}$$

The kind of distribution of charge carriers in a particle immediately before collision, established in the preceding Section, implies that at $t = 0$ the noticeable change of concentration n in terms of coordinates occurs in the particle surface layer of thickness d, with the characteristic length of variation of n along the normal to particle surface and along the latter are, respectively equal d and R. Hence when

$$\delta \ll d, \quad r_c \ll R \tag{3.8}$$

concentration n at the instant of time $t = 0$ inside the boundary layer of thickness $\sim\delta$ may be considered constant and equal to $n_s(E_0, e_p)$ at the contact point immediately before collision, as determined in the preceding Section. Consequently, neglecting the terms $\sim\delta / d, \sim r_c / R$, the initial conditions for system (3.5) can be specified thus:

$$n^* = 1, \quad t^* = 0 \tag{3.9}$$

Note that the first of inequalities (3.8) follows from conditions (3.4) and the second is always satisfied, if the particle sustains small deformations at impact.

From the second of Eqs. (3.5) and the second of boundary conditions (3.6) we have

$$\partial\varphi^* / \partial z^* = -e_r E_s \delta / (kT)$$

Substituting this expression for the derivative of φ^* into the first of equalities (3.5) and reverting to dimensional variables, we obtain the equation

$$\frac{\partial n}{\partial t} + V\frac{\partial n}{\partial z} - D\frac{\partial^2 n}{\partial z^2} = 0, \quad V \equiv \frac{e_r D E_s}{kT} \tag{3.10}$$

where V is the projection on the z-axis of the velocity of the ordered motion of charge carriers of the r variety induced by the electric field. The initial and boundary conditions for Eq. (3.10) are, as implied by equalities (3.6)—(3.9), of the form

$$n(0, z) = n_s, \quad n(t, \infty) = n_s, \quad n(t, 0) = n_w \tag{3.11}$$

The density of the electric charging current i flowing from the contact area to the particle is determined by the formula

$$i(t) = (-e_r D\, \partial n / \partial z + 2 e_r D n \varkappa E_s)_{z=0} = -e_r D(\partial n / \partial z)_{z=0} + 2 e_r D n_w \varkappa E_s, \quad \varkappa = e_r E_s / (2kT) \tag{3.12}$$

in which the second of equalities (3.1) and boundary conditions $n\,(n \equiv n_r)$ for concentration at the wall are taken into account.

Let us determine the derivative $\partial n / \partial z$ appearing in this equality by solving Eq. (3.10) using the Laplace transform. Let $N(p, z)$ be the image of function $n(t, z)$. Then if follows from Eq. (3.10) and conditions (3.11) that function $N(p, z)$ satisfies the relations

$$pN - n_s + VdN / dz - Dd^2N / dz^2 = 0, \quad N(p, 0) = n_w / p, \quad N(p, \infty) = n_s / p \tag{3.13}$$

The solution of problem (3.13) is of the form

$$N(p, z) = \frac{n_w - n_s}{p}\exp(-\lambda z) + \frac{n_s}{p}, \quad \lambda \equiv -\varkappa + \sqrt{\varkappa^2 + \frac{p}{D}}$$

We calculate the derivative $(dN / dz)_{z=0}$ and, using tables of Laplace inverse transforms, obtain $(dn / dz)_{z=0}$, and from equality (3.12) the final formula for the determination of current density

$$i(t) = e_r D\left[\varkappa(n_w + n_s) + \frac{n_w - n_s}{\sqrt{\pi D t}}\left(\exp(-D\varkappa^2 t) + \sqrt{\pi D \varkappa^2 t}\,\operatorname{erf}\sqrt{D\varkappa^2 t}\right)\right] \tag{3.14}$$

Let us consider the expression for the quantity δ in terms of the problem parameters. As indicated above, the concentration of charge carriers of the r variety on contact of the particle with the wall changes at the contact surface by $n_w - n_s$, as the result of their interaction with the wall. Owing to the diffusion process this perturbation propagate toward the inside of the particle. If there is no electromagnetic field $(E_s = 0)$ in the contact region, then obviously, the noticeable perturbation of concentration n at the instance of the particle-wall contact severance is concentrated in the diffusion boundary layer of thickness $\delta \sim \sqrt{D\tau}$ (we recall that the case when δ satisfies inequalities (3.4) is considered here). When $e_r E_s > 0$, the electric field contributes to the propagation of perturbations of concentration n from the contact area into the particle, since it acts on charge carriers of the r variety with a force that is oriented away from the wall. It follows directly from Eq. (3.10) that in this case the perturbation of concentration n would have propagated under the action of the electric field only up to the instance of collision end over a distance $V\tau = e_r D E_s \tau / (kT)$. For δ we, thus, have the estimate

$$\delta \sim \max\,(e_r D E_s \tau / (kT), \sqrt{D\tau}), \quad e_r E_s > 0 \tag{3.15}$$

If $e_r E_s < 0$, the electric field impedes the propagation of perturbations of concentration n from the contact surface into the particle (since a force directed toward the wall acts on charge carriers of the r variety) and, consequently, $\delta \leqslant \sqrt{D\tau}$. Moreover, in this case Eq. (3.10) implies that even when the collision duration is infinite $(\tau = \infty)$, the noticeable perturbation of concentration n is concentrated in the stationary $(\partial n / \partial t = 0)$ diffusion boundary layer, as $t \to \infty$. Its thickness δ_∞ can be determined without solving Eq. (3.10), using the condition that during the time τ' in which charge carriers of the r variety pass, under the action of the electric field force, the distance δ_∞, and their mean square shift produced by diffusion is exactly equal δ_∞^2, i.e.

$$\delta_\infty^2 = D\tau', \quad \tau' = \delta_\infty / |V|, \quad V = e_r D E_s / (kT)$$

In this case the final estimate of δ is obviously of the form

$$\delta \sim \min\left(\delta_\infty = kT / |e_r E_s|, \sqrt{D\tau}\right), \quad e_r E_s < 0 \tag{3.16}$$

When $E_s = 0$, estimates (3.15) and (3.16) are, as expected, equal, yielding $\delta \sim \sqrt{D\tau}$.

Formula (3.14) for $i(t)$ is simplified when $|\varkappa|\sqrt{D\tau} \ll 1$ or $|\varkappa|\sqrt{D\tau} \gg 1$. We have

$$\frac{i(t)}{e_r D} = \begin{cases} (n_w - n_s)/\sqrt{\pi D t}, & |\varkappa|\sqrt{D\tau} \ll 1 \\ 2\varkappa n_w, & \varkappa\sqrt{D\tau} \gg 1 \\ 2\varkappa n_s, & -\varkappa\sqrt{D\tau} \gg 1 \end{cases} \tag{3.17}$$

4. The expression for Δe_p. When function $i(t)$ is known, Δe_p can be obviously determined using formulas

$$\Delta e_p = \int_{\Sigma_c} \Delta q(r)\, ds, \quad \Delta q(r) = \int_0^{\tau(r)} i(t)\, dt \tag{4.1}$$

where Σ_c is that part of the particle surface (and its area) which is in contact with the wall at impact, ds is an element of surface Σ_c, $\Delta q(r)\, ds$ is the electric charge acquired by the particle as the result of contact with the wall over the surface element ds at point $r \in \Sigma_c$, and $\tau(r)$ is the duration of contact at point r. Formula (4.1) for Δe_p was derived on the assumption that the mechanical and electrical contacts between sections of particle and wall surfaces coincide. In this way the partial neutralization of the particle electric charge, due to gas discharge which can occur at particle rebound from the wall /9/, is neglected. This is valid for fairly small particles for which the striking of an arc is difficult /9/. The simplest way of taking into account the gas discharge at the instant of contact breaking is to assume that its effect limits the electric field intensity E_z by some limit quantity E_+. Taking into account the electric field generated by the charge and acquired by a particle, as the result of its contact with the wall, we obtain for the electric field intensity E_z the formula

$$E_z = E_w - 4\pi\varepsilon^{-1}\Delta q \tag{4.2}$$

where E_w is defined by equality (2.11). Formulas (4.1) hold for $|E_z| \leqslant E_+$, but, if their application to the determination of E_z results in the inequality $|E_z| > E_+$, it is necessary to substitute $(\mathrm{sign}\, E_z) E_+$ for E_z, after which Δq is obtained from (4.2) by elementary calculations.

The collision parameters Σ_c and $\tau(r)$ in the expression for Δe_p depend on the particle approach velocity v to the wall. For a normal elastic impact of a spherical particle they can be determined by the formulas /8/

$$\Sigma_c = \pi R h, \quad h = R\left[\frac{5\pi\rho v^2}{4}\left(\frac{1-\nu_p^2}{E_p^Y} + \frac{1-\nu_w^2}{E_w^Y}\right)\right]^{2/5}, \quad \tau(r) = \tau\left(1 - \frac{2}{\pi}\arcsin\frac{r^2}{Rh}\right), \quad \tau = \frac{3h}{v} \tag{4.3}$$

where ρ is the particle mass density, ν_p and E_p^Y (ν_w, E_w^Y) are, respectively, the Poisson coefficient and the Young modulus of the particle (wall), and r is the distance between the center of contact area and point $r \in \Sigma_c$. This shows that Δe_p depends on the particle approach velocity to the wall. The formulas obtained above for Δe_p and $i(t)$ indicate that this dependence can be fairly complex. The particle approach velocity (v) to the wall is determined by the flow of suspension as a whole. Velocity v can be calculated in many instances independently of the problem of charging particles. When the particle approach velocity to the wall is known, then, using the derived here formulas for Δe_p and formula (1.1), it is not difficult to determine the charging current at the boundaries.

As an example, let us consider the charging of aerosol particles of ice (hail, snow) by collision with the surface of a metal body moving in clouds or precipitations. For pure ice particles of binary ion conductance we have: $\varepsilon_p = 72$, $e = \pm 1.6 \cdot 10^{-19}$ C, $d = 10^{-6}$ m, $\sigma = 4 \cdot 10^{-7}$ ohm$\cdot$m^{-1}, $\tau_e = \varepsilon_p/(4\pi\sigma) = 1.6 \cdot 10^{-3}$ s, $D \simeq 4\pi\sigma d^2/\varepsilon_p = 6 \cdot 10^{-10}$ m^2/s, $n^\circ \simeq \varepsilon_p kT/(8\pi e^2 d^2) = 4 \cdot 10^{19}$ m^{-3}, $E_p^Y = 3 \cdot 10^9$ N/m^2 and $\nu_p = 0.3$.

In the eleastic collision of an aerosol ice particle with the surface of a metal body the following basic characteristic parameters obtained from (4.3) apply at impact (for $R = 10^{-4}$ m, and $v = 10$ m/s): $R/v = 10^{-5}$ s, $\tau = 9 \cdot 10^{-7}$ s, $h = 3 \cdot 10^{-6}$ m, $r_c = \sqrt{Rh} = 1.7 \cdot 10^{-5}$ m, $\Sigma_c = 9 \cdot 10^{-10}$ m^2, and $\sqrt{D\tau} = 2 \cdot 10^{-6}$ s. With their use we can readily verify that in the considered case inequalities (2.2), (3.4), and (3.8) are in fact satisfied, when $l_E \gg 10^{-2}$ m, $\tau_E \gg 10^{-3}$ s, and $\delta \sim \sqrt{D\tau}$. These relationships are satisfied, since length l_E is usually of the order of the radius of the body surface curvature (~ 1 m), and the time $\tau_E \sim l_E/v \sim 10^{-1}$ s. Moreover, for the above basic parameters we have $|eE_s|\sqrt{D\tau}/(2kT) \ll 1$, even when the electric field intensity attains its disruptive value of 10^6 V/m. Consequently, from formulas (3.15) and (3.16) we have $\delta \sim \sqrt{D\tau}$. Substituting in the integrals of (4.1) for $i(t)$ its first expression in (3.17) (since $|\varkappa|\sqrt{D\tau} \equiv |eE_s|\sqrt{D\tau}/(2kT) \ll 1$) and integrating with allowance for (4.3), we obtain

$$\Delta e_p = \frac{2C(\pi/2)}{\sqrt{\pi}} e(n_w - n_s)\Sigma_c \sqrt{D\tau} \tag{4.4}$$

where $C(\pi/2) = 0.78$ is the value of the Frenel integral, and n_s is that defined in Sect.2 (example 2). If there is no electric field on the body surface and the aerosol particles are not charged, then $n_s = n^\circ$, and for the obtained values of n°, Σ_c and $\sqrt{D\tau}$ for $n_w = 0$ and $e > 0$, we have $\Delta e_p = -10^{-16}$ C. As the result of charging aerosol particles, the charge of the metal body and the electric field intensity at its surface increase with the consequent decrease of concentration n_s in comparison with n° and some lowering of density of the electric current j flowing to the body. Formula (1.1) enables us to estimate the order of j. For example, when a body moves at velocity of 100 m/s in cumulonimbus clouds in which the aerosol particle concentration $n^- = 10^8$ m^{-3} and their radius $R = 10^{-4}$ m, we have $j \simeq -\Delta e_p n^- v^- = 10^{-6}$ A/m^2. Such currents are, in fact, observed in flights in clouds and precipitations.

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